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Difference Complex Projective Synchronization of the Fractional-Order 6-D chaotic Systems Using Active Control Technique

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Abstract In this paper, we investigate the combination difference complex projective synchronization (CDCPS) of the fractional-order (FO) 6-D chaotic system (CS) using active control technique. Based on the Lyapunov stability theory and active control technique is to achieve the combination difference synchronization between two master systems and one slave system. These master systems are FO 6-D Lü system, and the slave system is FO 6-D Liu system. The evolution of the difference synchronization scheme has additions to the unit of the existing synchronization technique. . Numerical simulations and graphical results are presented to demonstrate the effectiveness and reliability of CDCPS scheme. The presented CDCPS scheme has several applications in secure communications and information processing.

Keywords Combination difference synchronization · projective synchronization · 6-D chaotic system · active Control · fractional order

1 Introduction

The chaos synchronization and chaos control of dynamical systems are highly essential topics in mathematics and physics. A CS is a complex nonlinear system, which is extremely sensitive to initial states and parameters changes. Fowler et al.[1] are the first who introduced the complex Lorenz system in 1982, which performed an essential role in various branches of physics, mainly for secure communication. In complex variables, the unit of variables is getting twice which, enhance the protection of the transmitted information. After that, many CS and hyperchaotic (HC) complex systems are proposed and studied such as complex Lorenz, complex Chen, complex Lu system, complex HC Lorenz [2], etc. In FO, the complex CS and HC complex system also introduced in the various paper these are FO complex Lorenz, FO complex Liu [3], HC complex Lü [4], etc. Synchronization of complex CS (HC) has excellent application in the latest decades, such as complete synchronization [5], anti synchronization [6], difference synchronization [7, 8], function projective synchronization [9], projective synchronization (PS) [10], hybrid projective synchronization [11]. Mahmoud et al.[12] investigated with projective synchronization of the complex HC system and suggested a communication technique based on passive theory. In [13], author discussed the generalized combination complex synchronization for FO complex CS. In [4], the author investigated the complex dynamical behavior and modified PS in FO complex HC Lü system.

To obtain the chaos synchronization technique, several synchronization techniques have been produced such as active control [14], adaptive control [15], sliding mode control [16]. Among these techniques, the active control is used during combination difference complex synchronization. In difference synchronization technique is applied in chaotic secure communication to improve security data. In which the scaling matrix is designed as a complex matrix. Then it provides a complex and irregular transmitted signal that they have the powerful anti-attack capability and anti-translated ability compare with the previous model. The selection of FO complex HC system has been suggested for secure communication.

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The rest of the paper is organized as follow: Section 2 contain Preliminaries. In Section 3 consists the problem formulation of combination difference complex projective synchronization. Section 4 provide the system description of system and Section 5 contain Example of CDCPS. Section 6 contain the numerical simulation, Finally Section 7 contain conclusion.

2 Preliminaries

Definition 1: The Caputo's derivative for function $\Psi(t)$ with FO q is define by:

$$cD_t^q\Psi(y) = \frac{1}{\gamma(n-q)}\int_c^y \frac{\Psi^n(x)}{(y-x)^{q-n+1}}dx$$

where $n-1<q<n$ and $\gamma(q)=\int_0^\infty x^{q-1}e^{-x}dx$ is the gamma function.

In this paper, the Caputo definition is applied.

Property 1: If $\Psi_1(t)$ is a constant function and the order $q>0$,the Caputo FO derivative satisfies the conditions:

$$D^q\Psi_1(t)=0$$

Property 2: The Caputo fractional derivative satisfies the following linear property:

$$D^q[a_1\Psi_1(t)+a_2\Psi_2(t)] = a_1D^q\Psi_1(t)+a_2D^q\Psi_2(t)$$

where $\Psi_1(t)$ and $\Psi_2(t)$ are functions of t and a_1 and a_2 are constants.

3 Problem Formulation

Consider the two master chaotic (hyperchaotic) complex system as:

$$D^\alpha x = F(x)A+G(x) \tag{1}$$
$$D^\alpha y = F(y)A+G(y) \tag{2}$$

where $X=(x_1,x_2,...,x_n)^T$ and $Y=(y_1,y_2,...,y_n)^T$ are the state complex vector of system (1) and (2) respectively and $x=x^r+jx^i$, $y=y^r+jy^i$, $j=\sqrt{-1}$ and r represent as real parts and i represents imaginary parts. Assume $x_1=x_{11}+jx_{12}$,$x_2=x_{13}+jx_{14}$,....,$x_n=x_{1n-1}+jx_{1n}$, then $x^r=(x_{11},x_{13},...,x_{1n-1})^T$, $x^i=(x_{12},x_{14},...,x_{1n})^T$. $y_1=y_{11}+jy_{12}$,$y_2=y_{13}+jy_{14}$,....,$y_n=y_{1n-1}+jy_{1n}$, then $y^r=(y_{11},y_{13},...,y_{1n-1})^T$,$y^i=(y_{12}$,$y_{14},...,y_{1n})^T$. $F(x)$ and $F(y)$ are $n\times n$ complex matrix of state complex variables of (1) and (2). $G(x)$ and $G(y)$ are non-linear complex vector functions and A is an $n\times 1$ real (or complex) vectors of system parameter.

One slave CS (HC) complex system is written as

$$D^\alpha z = H(z)B+L(z)+u \tag{3}$$

where $z=(z_1,z_2,...,z_n)^T$ is a state complex vector of system (3). $z=z^r+jz^i$, $j=\sqrt{-1}$. Assume $z_1=z_{11}+jz_{12}$,$z_2=z_{13}+jz_{14}$,....,$z_n=z_{1n-1}+jz_{1n}$, then $z^r=(z_{11},z_{13},...,z_{1n-1})^T$, $z^i=(z_{12},z_{14},...,z_{1n})^T$. $H(z)$ is $n\times n$ complex matrix of state complex variables, $L(z)$ is a on-linear complex vector function. B is an $n\times 1$ real (or complex) vectors of system parameter. The controller is $u=u^r+ju^i$, where $u^r=(u_{11},u_{13},...,u_{1n-1})^T$, $u^i=(u_{12},u_{14},...,u_{1n})^T$.

Definition 1:[17, 18] For the maste complex CS (HC) systems (1) and (2) and the slave CS (HC) system (3) is said to be combination difference complex projective synchronization if there exist complex scaling matrice $\lambda\in R^{n\times n}$ such that.

$$lim_{t\to\infty}\|E(t)\| = lim_{t\to\infty}\|z-\lambda(x-y)\| = 0$$

where the complex scaling matrix $\lambda = h_1^r + jh_2^j = diag(h_{11}+jh_{12},h_{13}+jh_{14},...,h_{1n-1}+jh_{1n})$, $\|.\|$ present the norm of a matrix. $E(t)=E^r(t)+jE^i(t)$, where $E^r(t)=(E_{11},E_{13},...,E_{1n-1})^T$, $E^i=(E_{12},E_{14},...,E_{1n})^T$.

The error system is written as follows

$$\begin{aligned}
E(t) &= E^r(t)+jE^i(t)\\
&= z^r+jz^i-(h_1^r+jh_2^i)(x^r+jx^i-y^r-jy^i))\\
&= z^r+jz^i-(h_1^r+jh_2^i)(x^r-y^r+j(x^i-y^i))\\
&= z^r+jz^i-h_1^r(x^r-y^r)-jh_1^r(x^i-y^i)\\
&-jh_2^i(x^r-y^r)+h_2^i(x^i-y^i)\\
&= z^r-h_1^r(x^r-y^r)+h_2^i(x^i-y^i)\\
&+j(z^i-h_1^r(x^i-y^i)-h_2^i(x^r-y^r))
\end{aligned} \tag{4}$$

Separating real and imaginary error system, we get

$$E^r(t) = z^r-h_1^r(x^r-y^r)+h_2^i(x^i-y^i)$$
$$E^i(t) = z^i-h_1^r(x^i-y^i)-h_2^i(x^r-y^r)$$

The error dynamics is obtained as

$$\begin{aligned} D^qE^r(t) &= D^qz^r - h_1^r(D^qx^r - D^qy^r) + h_2^i(D^qx^i - D^qy^i) \\ &= H^r(z_{11}, z_{13}, ..., z_{1n-1})B + L^r(z_{11}, z_{13}, ... \\ &, z_{1n-1}) - h_1^r(F^r(x_{11}, x_{13}, ..., x_{1n-1})A + G^r(x_{11}, \\ &x_{13}, ..., x_{1n-1}) - F^r(y_{11}, y_{13}, ..., y_{1n-1})A \\ &- G^r(y_{11}, y_{13}, ..., y_{1n-1})) + h_2^i(F^i(x_{12}, x_{14} \\ &, ..., x_{1n})A + G^i(x_{12}, x_{14}, ..., x_{1n}) - F^i(y_{12}, y_{14} \\ &, ..., y_{1n})A - G^i(y_{12}, y_{14}, ..., y_{1n})) + u^r \end{aligned} \tag{5}$$

$$\begin{aligned} D^qE^i(t) &= D^qz^i - h_1^r(D^qx^i - D^qy^i) - h_2^i(D^qx^r - D^qy^r) \\ &= H^i(z_{12}, z_{14}, ..., z_{1n})B + L^i(z_{12}, z_{14} \\ &, ..., z_{1n}) - h_1^r(F^i(x_{12}, x_{14}, ..., x_{1n})A + G^i(x_{12} \\ &, x_{14}, ..., x_{1n}) - F^i(y_{12}, y_{14}, ..., y_{1n})A - G^i(y_{12} \\ &, y_{14}, ..., y_{1n})) + h_2^i(F^r(x_{11}, x_{13} \\ &, ..., x_{1n-1})A + G^r(x_{11}, x_{13}, ..., x_{1n-1}) - F^r(y_{11} \\ &, y_{13}, ..., y_{1n-1})A - G^r(y_{11}, y_{13}, ..., y_{1n-1})) + u^i \end{aligned} \tag{6}$$

Theorem 1: If the controller is constructed as the following form

$$\begin{aligned} u^r = &-H^r(z_{11}, z_{13}, ..., z_{1n-1})B - L^r(z_{11}, z_{13}, ..., z_{1n-1}) \\ &+ h_1^r(F^r(x_{11}, x_{13}, ..., x_{1n-1})A + G^r(x_{11}, x_{13}, ..., x_{1n-1}) \\ &- F^r(y_{11}, y_{13}, ..., y_{1n-1})A - G^r(y_{11}, y_{13}, ..., y_{1n-1})) \\ &- h_2^i(F^i(x_{12}, x_{14}, ..., x_{1n})A + G^i(x_{12}, x_{14}, ..., x_{1n}) \\ &- F^i(y_{12}, y_{14}, ..., y_{1n})AG^i(y_{12}, y_{14}, ..., y_{1n})) \\ &- K_1E^r(t) \end{aligned} \tag{7}$$

$$\begin{aligned} u^i = &-H^i(z_{12}, z_{14}, ..., z_{1n})B + L^i(z_{12}, z_{14}, ..., z_{1n}) \\ &+ h_1^r(F^i(x_{12}, x_{14}, ..., x_{1n})A + G^i(x_{12}, x_{14}, ..., x_{1n}) \\ &- F^i(y_{12}, y_{14}, ..., y_{1n})A - G^i(y_{12}, y_{14}, ..., y_{1n})) \\ &- h_2^i(F^r(x_{11}, x_{13}, ..., x_{1n-1})A + G^r(x_{11}, x_{13}, ..., x_{1n-1}) \\ &- F^r(y_{11}, y_{13}, ..., y_{1n-1})A - G^r(y_{11}, y_{13}, ..., y_{1n-1})) \\ &- K_2E^i(t) \end{aligned} \tag{8}$$

Then the slave system (3) can asymptotically synchronize the combination difference system between two master systems (1) and (2) with regard to the complex scaling matrix, where $K_1, K_2 > 0$ is a constant.

Proof. Using equations (7) and (8) are substitute in (5) and (6) respectively. The error systems (5) and (6) are reduced in the following form.

$$D^qE^r(t) = -K_1E^r(t) \tag{9}$$

$$D^qE^i(t) = -K_2E^i(t) \tag{10}$$

Selected a Lyapunov function as

$$V(E^r, E^i) = \frac{1}{2}((E^r)^TE^r + (E^i)^TE^i) \tag{11}$$

Then the derivative of V along the trajectories of the error dynamics (9) and (10) is takes the form:

$$\begin{aligned} D^qV(E^r, E^i) &= (D^qE^r)^TE^r + (D^qE^i)^TE \\ &= -K_1(E^r)^2 - K_2(E^i)^2 \\ &\leq 0 \end{aligned}$$

Since $V > 0$ and $D^qV < 0$ and thus from theorem 1, the error systems are asymptotically stable. □

4 System Discription of fractional order 6-D Lü and 6-D Liu system

Consider the[4] HC complex Lü systems describe in following manner:

$$\begin{cases} D^qx_1 = & a_1(x_2 - x_1) + x_4 \\ D^qx_2 = & c_1x_2 - x_1x_3 + x_4 \\ D^qx_3 = & \frac{1}{2}(\bar{x}_1x_2 + x_1\bar{x}_2) - c_1x_3 \\ D^qx_4 = & \frac{1}{2}(\bar{x}_1x_2 + x_1\bar{x}_2) - d_1x_4 \end{cases} \tag{12}$$

where $x = (x_1, x_2, x_3, x_4)^T$ is the state vector, $x_1 = x_{11} + ix_{12}$ and $x_2 = x_{13} + ix_{14}$ are complex state vector, and $x_3 = x_{15}$ and $x_4 = x_{16}$ are real state vector. a_1, b_1, c_1, d_1 are the parameters of the system (12).

$$\begin{cases} D^qx_{11} + iD^qx_{12} & = a_1(x_{13} + ix_{14} - (x_{11} + ix_{12})) \\ & +x_{16} \\ D^qx_{13} + iD^qx_{14} & = c_1(x_{13} + ix_{14)} - (x_{11} + ix_{12})x_{15} \\ & +x_{16} \\ D^qx_{15} & = \frac{1}{2}((x_{11} - ix_{12})(x_{13} + ix_{14}) + \\ & (x_{11} + ix_{12})(x_{13} - ix_{14})) - c_1x_{15} \\ D^qx_{16} & = \frac{1}{2}((x_{11} - ix_{12})(x_{13} + ix_{14}) + \\ & (x_{11} + ix_{12})(x_{13} - ix_{14})) - d_1x_{16} \end{cases} \tag{13}$$

Split real and imaginary parts of system (13), we get FO 6-D Lü system

$$\begin{cases} D^qx_{11} = & a_1(x_{13} - x_{11}) + x_{16} \\ D^qx_{12} = & a_1(x_{14} - x_{12}) \\ D^qx_{13} = & b_1x_{13} - x_{11}x_{15} + x_{16} \\ D^qx_{14} = & b_1x_{14} - x_{12}x_{15} \\ D^qx_{15} = & x_{11}x_{13} + x_{12}x_{14} - c_1x_{15} \\ D^qx_{16} = & x_{11}x_{13} + x_{12}x_{14} - d_1x_{16} \end{cases} \tag{14}$$

Similarly we obtain 6-D FO Liu system [3]

$$\begin{cases} D^q x_{21} = & -a_2 x_{21} - d_2(x_{23}^2 - x_{24}^2) \\ D^q x_{22} = & -a_2 x_{22} - 2d_2 x_{23} x_{24} \\ D^q x_{23} = & b_2 x_{23} - e_2(x_{21}x_{25} - x_{22}x_{26}) \\ D^q x_{24} = & b_2 x_{24} - e_2(x_{22}x_{25} + x_{21}x_{26}) \\ D^q x_{25} = & -c_2 x_{25} + f_2(x_{21}x_{23} - x_{22}x_{24}) \\ D^q x_{26} = & -c_2 x_{26} + f_2(x_{21}x_{24} + x_{22}x_{23}) \end{cases} \quad (15)$$

5 Numerical Example

The fractional-order 6-D Lü is considered as a first master system.

$$\begin{cases} D^q x_{11} = & a_1(x_{13} - x_{11}) + x_{16} \\ D^q x_{12} = & a_1(x_{14} - x_{12}) \\ D^q x_{13} = & b_1 x_{13} - x_{11}x_{15} + x_{16} \\ D^q x_{14} = & b_1 x_{14} - x_{12}x_{15} \\ D^q x_{15} = & x_{11}x_{13} + x_{12}x_{14} - c_1 x_{15} \\ D^q x_{16} = & x_{11}x_{13} + x_{12}x_{14} - d_1 x_{16} \end{cases} \quad (16)$$

The identical fractional-order 6-D Lü is considered as a second master system.

$$\begin{cases} D^q y_{11} = & a_1(y_{13} - y_{11}) + y_{16} \\ D^q y_{12} = & a_1(y_{14} - y_{12}) \\ D^q y_{13} = & b_1 y_{13} - y_{11}y_{15} + y_{16} \\ D^q y_{14} = & b_1 y_{14} - y_{12}y_{15} \\ D^q y_{15} = & y_{11}y_{13} + y_{12}y_{14} - c_1 y_{15} \\ D^q y_{16} = & y_{11}y_{13} + y_{12}y_{14} - d_1 y_{16} \end{cases} \quad (17)$$

The fractional-order 6-D Liu system is considered as a slave system.

$$\begin{cases} D^q z_{11} = & -a_2 z_{11} - d_2(z_{13}^2 - z_{14}^2) + u_{11} \\ D^q z_{12} = & -a_2 z_{12} - 2d_2 z_{13} z_{14} + u_{12} \\ D^q z_{13} = & b_2 z_{13} - e_2(z_{11}z_{15} - z_{12}z_{16}) + u_{13} \\ D^q z_{14} = & b_2 z_{14} - e_2(z_{12}z_{15} + z_{11}z_{16}) + u_{14} \\ D^q z_{15} = & -c_2 z_{15} + f_2(z_{11}z_{13} - z_{12}z_{14}) + u_{15} \\ D^q z_{16} = & -c_2 z_{16} + f_2(z_{11}z_{14} + z_{12}z_{13}) + u_{16} \end{cases} \quad (18)$$

Error states are given by:

$$\begin{cases} E_1 = z_1 - h_1(x_1 - y_1) \\ E_2 = z_2 - h_2(x_2 - y_2) \\ E_3 = z_3 - h_3(x_3 - y_3) \end{cases} \quad (19)$$

where $E_1 = (E_{11} + iE_{12})$, $E_2 = (E_{13} + iE_{14})$, $E_3 = E_{15} + iE_{16}$

$$\begin{cases} E_{11} = & z_{11} - h_{11}(x_{11} - y_{11}) + h_{12}(x_{12} - y_{12}) \\ E_{12} = & z_{12} - h_{11}(x_{12} - x_{22}) - h_{12}(x_{11} - y_{11}) \\ E_{13} = & z_{13} - h_{13}(x_{13} - y_{13}) + h_{14}(x_{14} - y_{14}) \\ E_{14} = & z_{14} - h_{13}(x_{14} - y_{14}) - h_{14}(x_{13} - y_{13}) \\ E_{15} = & z_{15} - h_{15}(x_{15} - y_{15}) + h_{16}(x_{16} - y_{16}) \\ E_{16} = & z_{16} - h_{15}(x_{16} - y_{16}) - h_{16}(x_{15} - y_{15}) \end{cases} \quad (20)$$

Then the control function is selected as:

$$\begin{cases} u_{11} = & a_2 z_{11} + d2(z_{13}^2 - z_{14}^2) + h_{11}(a_1(x_{13} - x_{11}) \\ & +x_{16} - a_1(y_{13} - y_{11}) - y_{16}) - h_{12}(a_1(x_{14} \\ & -x_{12}) - a_1(y_{14} - y_{12})) - K_1 E_{11} \\ u_{12} = & a_2 z_{12} + 2d_2 z_{13} z_{14} + h_{11}(a_1(x_{14} - x_{12}) \\ & -a_1(y_{14} - y_{12})) + h_{12}(a_1(x_{13} - x_{11}) \\ & +x_{16} - a_1(y_{13} - y_{11}) - y_{16}) - K_2 E_{12} \\ u_{13} = & -b_2 z_{13} + e_2(z_{11}z_{15} + z_{12}z_{16}) + h_{13}(b_1 x_{13} \\ & -x_{11}x_{15} + x_{16} - b_1 y_{13} + y_{11}y_{15} - y_{16}) \\ & -h_{14}(b_1 x_{14} - x_{12}x_{15} - b_1 y_{14} + y_{12}y_{15}) \\ & -K_3 E_{13} \\ u_{14} = & -b_2 z_{14} + e_2(z_{12}z_{15} + z_{11}z_{16}) + h_{13}(b_1 x_{14} \\ & -x_{12}x_{15} - b_1 y_{14} + y_{12}y_{15}) + h_{14}(b_1 x_{13} \\ & -x_{11}x_{15} + x_{16} - b_1 y_{13} + y_{11}y_{15} - y_{16}) \\ & -K_4 E_{14} \\ u_{15} = & c_2 z_{15} - f_2(z_{11}z_{13} + z_{12}z_{14}) + h_{15}(x_{11}x_{13} \\ & +x_{12}x_{14} - c_1 x_{15} - y_{11}y_{13} - y_{12}y_{14} + c_1 y_{15}) \\ & -h_{16}(x_{11}x_{13} + x_{12}x_{14} - d_1 x_{16} - y_{11}y_{13} \\ & -y_{12}y_{14} + d_1 y_{16}) - K_5 E_{15} \\ u_{16} = & c_2 z_{16} + f_2(z_{11}z_{14} + z_{12}z_{13}) + h_{15}(x_{11}x_{13} \\ & +x_{12}x_{14} - d_1 x_{16} - y_{11}y_{13} - y_{12}y_{14} + d_1 y_{16}) \\ & +h_{16}(x_{11}x_{13} + x_{12}x_{14} - c_1 x_{15} - y_{11}y_{13} \\ & -y_{12}y_{14} + c_1 y_{15}) - K_6 E_{16} \end{cases} \quad (21)$$

The error dynamical system are given in the form

$$D^q E_{11} = -K_1 E_{11}, D^q E_{12} = -K_2 E_{12}, D^q E_{13} = -K_3 E_{13}$$
$$D^q E_{14} = -K_4 E_{14}, D^q E_{15} = -K_5 E_{15}, D^q E_{16} = -K_6 E_{16} \quad (22)$$

Thus the system achieves the combination difference complex projective synchronization in complex system.

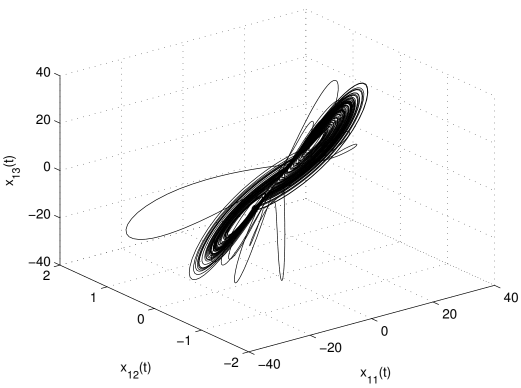
6 Simulation and Results

The parameter values of master and slave systems are chosen as ($a_1 = 42$, $b_1 = 25$, $c_1 = 6$, $d_1 = 5$), ($a_2 = 1$,

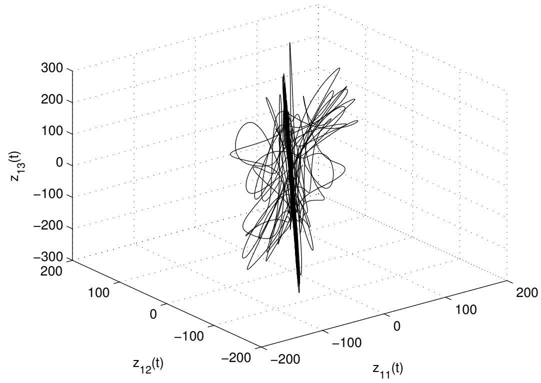
$b_2 = 5/2$, $c_2 = 5$, $d_2 = 1$, $e_2 = 4$, $f_2 = 4$) for system (16-18). The initial conditions of the master systems (16),(17) and the slave system (18) are respectively taken as. $(x_{11}, x_{12}, x_{13}, x_{14}, x_{15}, x_{16}) = (-0.4084$, 0.0492, -1.8907, 0.1506, 49.2122, -0.2999),(y$_{11}, y_{12}, y_{13}$,y$_{14}, y_{15}, y_{16}) =$ $(-0.5090, 0.0500, -2.001, 0.2500, 50.200$, -0.3)(z$_{11}, z_{12}, z_{13}, z_{14}, z_{15}, z_{16}) = (21.5257, -0.1541$, 23.3017, -0.1031, -0.1057, -0.0135)$of order$q=0.99.$Initial states for the error system$(E$_{11}, E_1$ = (19.6885, -0.4543, 22.4629, -0.1467, 4.8335, 1.9616).$The scaling complex matrix is chosen as \lambda =$ $diag(h_{11} + ih_{12}, h_{13} + ih_{14}, h_{15} + ih_{16}) =$ $(2 + i3, 4 + i4, 5 + i2)$.3-D phase portrait of FO 6-D Lü system and 6-D Liu system are depicted in Fig.1(a-b). CDCPS trajectories are illustrated in Fig.2(a-f) and the error converge to zero as shown in Fig.2(g)

7 Conclusion

We proposed a CDCPS technique for the FO 6-D chaotic systems where the difference among the state variables of two master systems synchronizes with the state variables of one slave system. Furthermore, this scheme is based on Lyapunov stability theory, and the nonlinear controllers are constructed to perform the difference complex synchronization. The general proofs and simulations are presented to illustrate the efficacy and feasibility of the suggested active control technique.

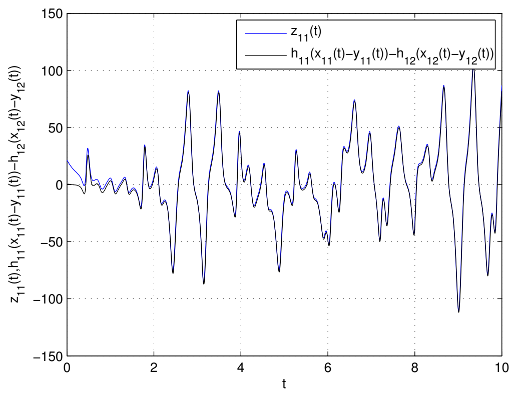


(a)

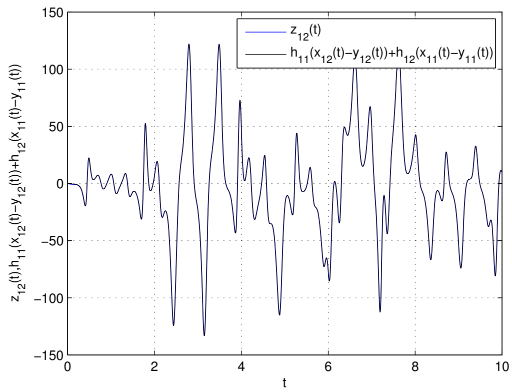


(b)

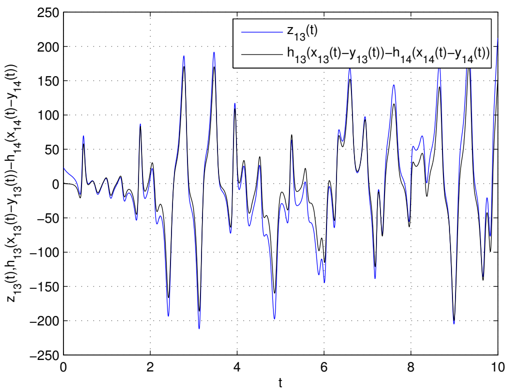
Fig 1: 3-D phase portrait of FO 6-D Lü system and 6-D Liu system in (a)$x_{11} - x_{12} - x_{13}$ and (b) $z_{11} - z_{12} - z_{13}$



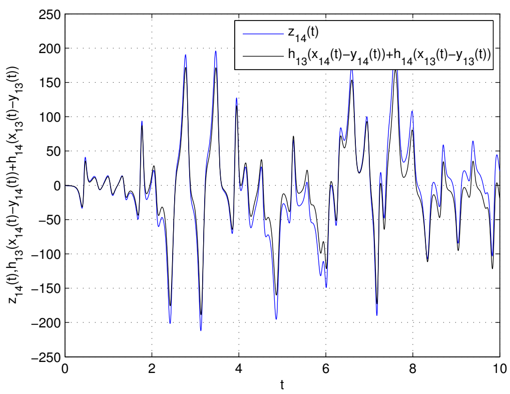
(a)



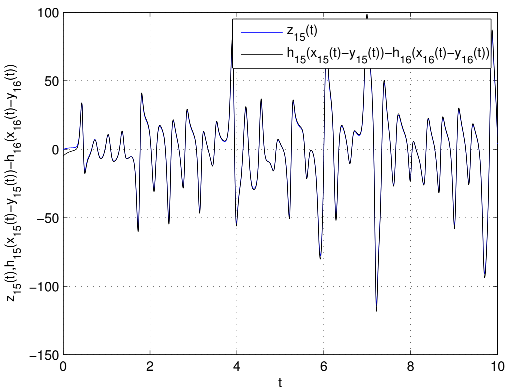
(b)



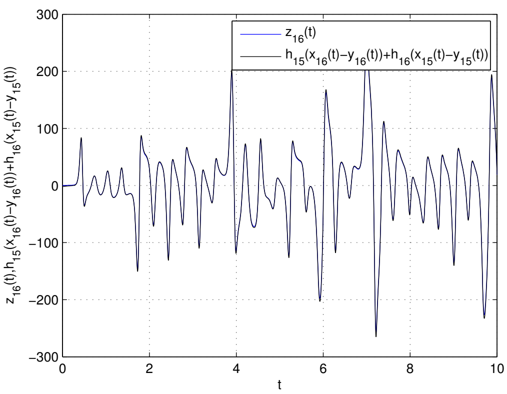
(c)



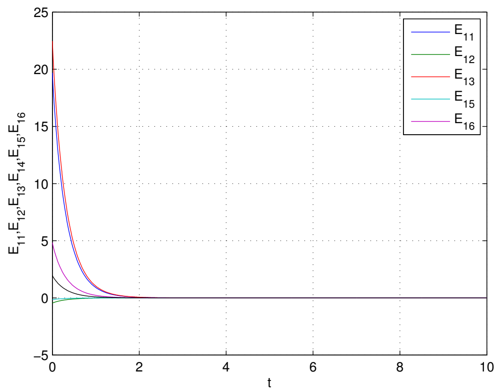
(d)



(e)



(f)



(g)

Fig 2: Combination difference complex projective synchronization between (a) $z_{11}(t)$ and $h_{11}(x_{11} - y_{11}) - h_{12}(x_{12} - y_{12})$, (b) $z_{12}(t)$ and $h_{11}(x_{12} - y_{12}) + h_{12}(x_{11} - y_{11})$, (c) $z_{13}(t)$ and $h_{13}(x_{13} - y_{13}) - h_{14}(x_{14} - y_{14})$,(d)$z_{14}(t)$ and $h_{13}(x_{14} - y_{14}) + h_{14}(x_{13} - y_{13})$,(e)$z_{15}(t)$ and $h_{15}(x_{15} - y_{15}) - h_{16}(x_{16} - y_{16})$,(f)$z_{16}(t)$ and $h_{15}(x_{16} - y_{16}) + h_{16}(x_{15} - y_{15})$, (g) CDCPS error converging to 0

References

1. AC Fowler, JD Gibbon, and MJ McGuinness. The complex lorenz equations. *Physica D: Nonlinear Phenomena*, 4(2):139–163, 1982.
2. Gamal M Mahmoud, Mansour E Ahmed, and Emad E Mahmoud. Analysis of hyperchaotic complex lorenz systems. *International Journal of Modern Physics C*, 19(10):1477–1494, 2008.
3. Gamal M Mahmoud, Tarek M Abed-Elhameed, and Mansour E Ahmed. Generalization of combination–combination synchronization of chaotic n-dimensional fractional-order dynamical systems. *Nonlinear Dynamics*, 83(4):1885–1893, 2016.
4. Li-xin Yang and Jun Jiang. Complex dynamical behavior and modified projective synchronization in fractional-order hyper-chaotic complex lü system. *Chaos, Solitons & Fractals*, 78:267–276, 2015.
5. Gamal M Mahmoud and Emad E Mahmoud. Complete synchronization of chaotic complex nonlinear systems with uncertain parameters. *Nonlinear Dynamics*, 62(4):875–882, 2010.
6. Ping Liu and Shutang Liu. Anti-synchronization between different chaotic complex systems. *Physica Scripta*, 83(6):065006, 2011.
7. Vijay K Yadav, Vijay K Shukla, and Subir Das. Difference synchronization among three chaotic systems with exponential term and its chaos control. *Chaos, Solitons & Fractals*, 124:36–51, 2019.
8. Ayub Khan and Pushali Trikha. Compound difference anti-synchronization between chaotic systems of integer and fractional order. *SN Applied Sciences*, 1(7):757, 2019.
9. Shutang Liu and Fangfang Zhang. Complex function projective synchronization of complex chaotic system and its applications in secure communication. *Nonlinear Dynamics*, 76(2):1087–1097, 2014.
10. Zhaoyan Wu, Jinqiao Duan, and Xinchu Fu. Complex projective synchronization in coupled chaotic complex

dynamical systems. *Nonlinear Dynamics*, 69(3):771–779, 2012.

11. Hadi Delavari and Milad Mohadeszadeh. Hybrid complex projective synchronization of complex chaotic systems using active control technique with nonlinearity in the control input. *Journal of Control Engineering and Applied Informatics*, 20(1):67–74, 2018.
12. Gamal M Mahmoud, Emad E Mahmoud, and Ayman A Arafa. On projective synchronization of hyperchaotic complex nonlinear systems based on passive theory for secure communications. *Physica Scripta*, 87(5):055002, 2013.
13. Cuimei Jiang, Shutang Liu, and Da Wang. Generalized combination complex synchronization for fractional-order chaotic complex systems. *Entropy*, 17(8):5199–5217, 2015.
14. Sachin Bhalekar. Synchronization of non-identical fractional order hyperchaotic systems using active control. *World Journal of Modelling and Simulation*, 10(1):60–68, 2014.
15. MT Yassen. Adaptive control and synchronization of a modified chua's circuit system. *Applied Mathematics and Computation*, 135(1):113–128, 2003.
16. Sundarapandian Vaidyanathan. Hybrid chaos synchronization of rikitake two-disk dynamo chaotic systems via adaptive control method. *International Journal of ChemTech Research*, 8(11):12–25, 2015.
17. Eric Donald Dongmo, Kayode Stephen Ojo, Paul Woafo, and Abdulahi Ndzi Njah. Difference synchronization of identical and nonidentical chaotic and hyperchaotic systems of different orders using active backstepping design. *Journal of Computational and Nonlinear Dynamics*, 13(5):051005, 2018.
18. Junwei Sun, Guangzhao Cui, Yanfeng Wang, and Yi Shen. Combination complex synchronization of three chaotic complex systems. *Nonlinear dynamics*, 79(2):953–965, 2015.